UNIT 6: EXPONENTIAL EQUATIONS & FUNCTIONS

Part A: Video Tutorial Section

Video 1:

https://www.youtube.com/watch?v=CJOKV1noths (**Review how to use a graphic calculator to find exponents and square roots)**

Video 2 and 3:

https://www.youtube.com/watch?v=PXdYQqjHAuE **(Properties of Square Roots)**

https://www.youtube.com/watch?v=W7juESNGi6k **(More Examples of Properties of Square Roots)**

Video 4 and 5:

https://www.youtube.com/watch?v=M_zljMICB3k **(Simplifying Square Roots)**

https://www.youtube.com/watch?v=CN1I81Suuks **(More Examples of Simplifying Square Roots)**

Video 6 and 7:

https://www.youtube.com/watch?v=XX_MW4fVeTM **(Factoring Square Roots with Variables)**

https://www.youtube.com/watch?v=4jhG7g5qgO0 **(Factoring Square Roots with Variables)**

Video 8

https://www.youtube.com/watch?v=Zt2fdy3zrZU **(BASIC Review of Rules of Exponents)**

Videos 9 and 10:

https://www.youtube.com/watch?v=-TpiL4J_yUA **(Product of Powers Property Exponents)**

https://www.youtube.com/watch?v=KwRtepnMnCM **(More Examples of Powers Property Exponents)**

Videos 11 and 12:

https://www.youtube.com/watch?v=tvj42WdKlH4 **(Quotient of Powers Property Exponents)**

https://www.youtube.com/watch?v=4bCfv7GGVac **(More Examples of Quotient of Powers Property**) Exponents)

Videos 13 and 14:

https://www.youtube.com/watch?v=4GfACnQTrFo **(Writing Expressions in Radical Form)**

https://www.youtube.com/watch?v=KBP5EKcG1s4 **(Writing Expressions in Rational Form)**

Video 15:

https://www.youtube.com/watch?v=ePKiIlv2IZA **(Using a Graphing Calculator to Solve)**

Video 16:

https://www.youtube.com/watch?v=6WMZ7J0wwMI **(Introduction to Exponential Functions)**

Video 17:

https://www.youtube.com/watch?v=PEtIQqvIoGU **(Basics You Need to Understand About Functions)**

Videos 18 and 19:

https://www.youtube.com/watch?v=8OHEgD6YMBw **(Linear Vs. Exponential Functions)**

https://www.youtube.com/watch?v=y2OHGJmMalc **(More Examples of Linear Vs. Exponential Functions)**

Video 20:

https://www.youtube.com/watch?v=oVUJMVR1JpA **(Solving for X, When X is an Exponent)**

Videos 21 and 22:

https://www.youtube.com/watch?v=vlSEpdWf2hQ **(Graphing Exponential Functions)**

https://www.youtube.com/watch?v=xZXvqycUTc8 **(Graphing Exponential Functions Using Graphing Calculator)**

Videos 23, 24, 25:

https://www.youtube.com/watch?v=z901HPbi964 **(Exponential Growth and Decay)**

https://www.youtube.com/watch?v=6WMZ7J0wwMI **Exponential Growth:**

https://www.youtube.com/watch?v=Yy0WeHG2zHc **(Exponential Decay)**

Videos 26 and 27:

https://www.youtube.com/watch?v=pXo0bG4iAyg **Geometric Sequences)**

https://www.youtube.com/watch?v=-5kIBPR2Npk **(More Examples for geometric Sequences)**

Video 28:

https://www.youtube.com/watch?v=EU0c6qrrevA **(Extending Geometric Sequences)**

Videos 29 and 30

https://www.youtube.com/watch?v=2w-McCDsgoE **(Graphing Geometric Sequences)**

https://www.youtube.com/watch?v=jU6JTdhHVuI **(More Examples of Graphing a Geometric Sequence)**

Video 31:

https://www.youtube.com/watch?v=bguje4yGTK0 **(Recursively Defined Sequences)**

Videos 32 and 33:

https://www.youtube.com/watch?v=IFHZQ6MaG6w **(Recursive Formulas for Sequences)**

https://www.youtube.com/watch?v=IEyojX9jzv0 **(More Examples of Recursive Formulas for Sequences)**

Video 34 and 35:

https://www.youtube.com/watch?v=2QxrpxXQX9k**(Translating Between Recursive Rules & Explicit Rules)**

https://www.youtube.com/watch?v=2QxrpxXQX9k **(More Examples of Translating Between Recursive Rules & Explicit Rules)**

Video 36:

https://www.youtube.com/watch?v=Iq7a2vEsT-o **(Writing Recursive Rules for Other Sequences)**

https://www.youtube.com/watch?v=vhxMgAGvD4M (**More Examples for Writing Recursive Rules for Other Sequences)**

Part B : Vocabulary, Hints and Explanations

Important Vocabulary That Students need to Understand!

closed	when the operation performed on any two numbers in the set results in a number that is also in the set
Nth root	when b^n = a for an integer *n* greater than 1 The nth root of a number may be real or imaginary numbers.
Exponential function	A function of the form y = $ab^{x,\text{ where a}}$
exponential growth	occurs when a quantity increases by the same factor over equal intervals of time
exponential groth function	A function in the form of: a(1+ $r^{)1,}$ where a > 0 and r > 0
compound interest	interest earned on the principal and on previously earned interest.
exponential decay	when a quantity decreases by the same factor over equal intervals of time
exponential decay function	A function of the form: y = $a(1-r)^1$ where a > 0 and 0< r<1
geometric sequence	The ratio between consecutive terms
common ratio	The ratio found in a geometric sequence
recursive rule	Gives the beginning term (s0 of a sequence and an equation that indicates how any term a_n in the sequence relates to the previous term

Some students struggle to recall that exponents mean repeated multiplication of the base number and NOT the base number and exponent multiplied together.

A student many think that $5^{2\,=}$ 5(2) 5^2 = 25 not 5(2) = 10

Hint: Tell the student that exponents may be tiny but they are powerful (like bees), they make numbers grow big fast (just like a bee sting makes your hand swell up fast)

Making sure that the student understands the concept of exponents before working with exponents in functions is important.

Hint: A student may relate or understand exponents for squares if he sees a multiplication table and draws a line through the perfect squares. This can also help when factoring square roots.

Properties of square roots:

A student needs to understand that when working with square roots the "hard operations" apply but not the "easy operations". If a student thinks of addition and subtraction as the "easy operations" (because they have been doing those since Elementary School), they can say that those operations do not apply to something "hard" like square roots.

(Note- on line version of word will not put in the square root or radical sign. When creating examples, I had to use the words "square root". When teaching you will want to use the symbol.)

Ex: square root of 36 + 64 ≠ square root of 36 + square root of 64

Square root of 100 ≠ 6 + 8

10 ≠ 14

With that same reasoning, the "harder operations" which are multiplication and division do apply to "hard math" such as square roots.

Ex: Ex. Square root of 4 • 9 = square root of 4 • square root of 9

Square root of 36 = 2 • 3

6 = 6

Simplifying a Square Root Expression: A student needs to think in terms of factoring a radical (another name for square root) into perfect squares whenever possible.

Ex: = square root of 150 = square root of 25 • square root of 6 (25 is a perfect square - 5 5 = 25)

Square root of 150 = 5 • square root of 6

Properties of Exponents:

Multiplying with powers (exponents) - when you multiply powers with the same base number (or variable), keep the base and add the exponents

Ex: $4^3 \cdot 4^2 = 4^{3+2} = 4^5$

A student may want to multiply the bases and add the exponents. Have the student complete

$4^3 = 64$

$4^2 = 16$

$64 + 16 = 80$

$16^5 = 1{,}048{,}576$

When a student breaks the equation into the component parts, he may see the error in multiplying the bases.

Dividing with powers (exponents) - when you divide powers with the same base number (or variable), keep the base and subtract.

Hint: A student is pairing up operations. The "get larger" operations of multiplication and addition go together. The "get smaller" operations of division and subtraction go together.

Negative Powers: A negative power "flips" you out!

Ex: $4^{-2 =}$ $1/4^2 = 1/16$

To the power of one: Remind students that a base without an exponent is that base to the power of one.

Ex: $5^2 \bullet 5 = 5^{(2+1)} = 5^3$

To the power of zero: Any base to the power of zero equals one

Ex: $123{,}456^0 = 1$

Radicals and Rational Exponents

The rule states WHEN $b^n = a$ for an integer >1, b is the n^{th} root of a (say what??!!??) The following examples may help you understand. Also, this is a calculator job or a guess and check!

Ex: $4^3 = 64$ SO - ^{3}square root of 64 = square root of 4•4•4

The n^{th} root can be expressed as fractional power.

Ex: $64^{1/3} = {}^3$square root of 64

Hint: Fractional exponents make answers smaller while whole number exponents make things larger. T
$4^{2=}$ 4• 4 = 16

$4^{1/2}$ = (square root) 4 = 2

A student can use the properties of exponents (from above) to simplify expressions involving rational exponents.

Ex: $16^{3/4} = (16^{1/4})^3$ rewrite the exponent such that one over the denominator becomes the exponent (base and exponent in parenthesis) to the power of the numerator

$= 2^3$ find the fourth root of 16

$= 8$ solve for the cube of 2

Hint: the numerator navigates to outside

Exponential Functions

An **exponential function** is a non-linear function where the y increases by exponentially (repeated multiplication) versus addition. Think of bacteria growing on a surface!

Formula: $y = ab^x$

To find the y-intercept in exponential function, substitute zero for the x (which is the exponent).

In $\mathbf{y=ab^x}$ the y value changes by factors of b as the x values increase by 1.

Ex:

x	0	1	2	3
y	2	10	50	1250

In this example x is increasing by 1, while y is changing by a factor of times 5.

The equation for this example is: $y = 2(5)^x$

The graph of an exponential function is a smooth, curved line.

Exponential Growth and Decay

Exponential growth occurs when a quantity increases by the same factor over equal intervals of time.

Please note the formula on the reference sheet for the Algebra 1 Regents is **NOT** the formula that Algebra 1 students should use! Students will need to memorize the formula for exponential growth and decay.

Exponential growth formula:

$y = a(1 + r)^t$

y = final amount

a = initial amount

r = rate of growth (in decimal form)

t = time

(1 + r) = the growth factor

Exponential decay formula:

$y = a(1 - r)^t$

y = final amount

a = initial amount

r = rate of growth (in decimal form)

t = time

(1 - r) = the decay factor

Compound Interest

Simple interest is interest earned on principal (or initial amount) only.

Compound interest is interest earned on principal AND previously earned interest.

Hint: explain a savings account to a student by demonstrating a simple interest versus a compound interest. Demonstrate that an initial savings account (or investment) of $500 at 5% interest for 5 years using both simple interest and compound interest.

Simple interest: I=prt

I = 500(.05)(5)

I = $125

500 + 125 = $625

Compound interest (without the formula)

Year one I = 500 (.05) = 25

Year two I = 525 (.05) = 26.25

Year three I = 551.25 (.05) = 27.56

Year three I = 578.81 (.05) = 28.94

Year four I = 607.75 (.05) = 30.39

Year five I = 638.14 (.05) = 31.91

With compound interest – after five years you have $670.05

The formula for compound interest is:

$y= P(I+ r/n)^{nt}$

P = principal (initial amount)

r = annual interest rate

t= time in years

n = number of times interest is compounded per year

In the above example the interest was compounded annually. If interest is compounded every 6 months the n would equal 2 times per year.

Geometric Sequence

In Unit 5 students learned to solve arithmetic sequences where each term in the sequence was found by adding the same intervals.

In a geometric sequence, each term in the sequence is found by multiplying the previous term by a common ratio.

Ex:

Position	1	2	3	4	5	6	7
Term	3	(3• 2) = 6	(6•2)= 12	(12•2) = 24	(24•2) = 48	(48•2) = 96	(96•2) = 192

Formula for geometric sequence: This formula **IS** in the Algebra 1 Reference Sheet and **CAN** be used by Algebra 1 students to solve geometric sequence for a desired term.

Formula: $a_n = a_1 r^{(n-1)}$

a_n is the desired term in the sequence

a_1 is the first term in the sequence

r is the common ration (what you are multiplying by)

Recursively Defined Sequences

The formulas learned above for arithmetic and geometric sequences were *explicit* equations.

A recursive rule gives the beginning term and an equation that indicates how many terms a_n in the sequence relates to the previous term.

When writing a recursive rule, the student writes the first term, the rule.

Ex; $a_1 = 3$, $a_n = a_{n-1} + 3$ for arithmetic sequences

$a_1 = 1$, $a_n = 3a_{n-1}$ for geometric sequences

The recursive equation for an arithmetic sequence is:

$a_n = a_{n-1} + d$, where d is the common difference

A student may be required to write the terms in a sequence and graph.

Ex: Write terms in a sequence :

Given: $a_1 = 2$, $a_n = a_{n-1} + 3$

Each term that follows would be:

$a_1 = 2$

$a_2 = a_1 + 3 = 5$

$a_3 = a_2 + 3 = 8$

$a_4 = a_3 + 3 = 11$

The student then graphs the coordinates:

(1,2) (2,5) (3,8) (4,11)

A student may be required to write the equation in recursive format from a chart.

Ex: Given a chart

position	1	2	3	4	5
term	3	8	13	18	23

The sequence is arithmetic with the first term 3 and the common difference 5.

Recursive equation for an arithmetic sequence is:

$a_n = a_{n-1} + d$ (substitute 5 for the d)

The equation for this table is:

$a_n = a_{n-1} +5$

Therefore, the recursive rule is:

$a_1 = 3$, $a_n = a_{n-1} + 5$

Translating Recursive Rules into Explicit Equations for Arithmetic Sequences:

Given a recursive rule:

$a_1 = 25$, $a_n = a_{n-1} + 10$

The first stated term is 25 and the common difference is 10

Recall the equation for an arithmetic sequence is:

$a_n = a_1 + (n-1)\ d$

Substitute 25 for the first term, and 10 for d (common difference)

$a_n = 25 + (n-1)10$

$a_n = 25 + 10n – 10$ distribute

$a_n = 10n + 15$ simplify

Translating Explicit Equations into Recursive Rules for Arithmetic Sequences:

Given an explicit equation:

$a_n = -2n + 3$

The equation represents a sequence with the first term –2(1) + 3 = 1, with a common difference of –2

Recall the recursive equation is:

$a_n = a_{n-1} + d$

Substitute –2 for d

$a_n = a_{n-1} + (-2)$

The recursive rule becomes:

$a_n = 1$, $a_n = a_{n-1} - 2$

The recursive equation for a geometric sequence is:

$a_n = r \bullet a_{n-1}$, where r is the common ratio

The student may be required to write terms in a sequence and graph.

Ex: Write the terms in a sequence

Given $a_1 = 1$, $a_n = 3a_{n-1}$

Each term that follows would be:

$a_1 = 1$

$a_2 = 3a_{n-1} = 3(1) = 3$

$a_3 = 3a_2 = 3(3) = 9=$

$a_4 = 3(9) = 27$

$a_5 = 3(27) = 81$

The student then graphs the coordinates:

(1,1) (2,3) (3,9) (4,27), (5,81)

A student may be required to write the equation in recursive format from a chart.

Ex: Given a chart

position	1	2	3	4	5
term	5	10	20	40	80

The sequence is geometric with the first term 5 and the common ratio 2.

Recall the recursive equation for a geometric sequence is:

$a_n = r\bullet a_{n-1}$

Substitute 2 for the common ratio , r

$a_n = 2a_{n-1}$

Therefore, the recursive rule is:

$a_n = 5,\ a_n = 2a_{n-1}$

Translating Recursive Rules into Explicit Equations for Geometric Sequences:

Given a recursive rule:

$a_1 = 3,\ \ a_n = 2a_{n-1}$

The first term is 3 and the common ratio is 2.

Recall the equation for a geometric sequence is:

$a_n = a_1 r^{n-1}$

Substitute 3 for a_1 and 2 for r:

$a_n = 3(2)^{n-1}$

Translating Explicit Equations into Recursive Rules for Geometric Sequences:

Given an explicit equation:

$a_n = -3(2)^{n-1}$

The equation represents a geometric sequence with the first term of –3 and a common ration of 2.

Recall the recursive equation is:

$a_n = r \bullet a_{n-1}$

Substitute 2 for the common ratio, r

$a_n = 2a_{n-1}$

Therefore, the recursive rule is:

$a_1 = -3$, $a_n = 2a_{n-1}$