# UNIT 7: POLYNOMIAL EQUATIONS & FACTORING

## Part A: Video Tutorial Section

Video 1:

https://www.youtube.com/watch?v=ffLLmV4mZwU **(The BASICS of Polynomials You Need to Know)**

Videos 2: and 3

https://www.youtube.com/watch?v=ZgFXL6SEUiI **(Adding/Subtracting Polynomials)**

https://www.youtube.com/watch?v=ZGl2ExHwdak (**More Examples of (Adding/Subtracting Polynomials)**

Video 4 and 5:

https://www.youtube.com/watch?v=BbaCM1S70bA **(Multiplying Polynomials FOIL METHOD)**

https://www.youtube.com/watch?v=bn51ML8jkIs **(More Examples of Multiplying Polynomials FOIL METHOD)**

Videos 6 and 7:

https://www.youtube.com/watch?v=wQU-OxIySTE **(Multiplying Polynomials Using BOX METHOD)**

https://www.youtube.com/watch?v=yZLMqkczZCE **(More Examples of BOX METHOD Example)**

Videos 8 and 9:

https://www.youtube.com/watch?v=_WGdcgFy6lI **(Special Products of Polynomials)**

https://www.youtube.com/watch?v=SlRgONK-DXA **(More examples of Special Products of Polynomials)**

Video 10 and 11:

https://www.youtube.com/watch?v=XRIWk25-cO4 **(Solve Polynomial Equations in Factored Form)**

https://www.youtube.com/watch?v=Q3LI11e19kc **(More Examples of Solving Polynomial Equations in Factored Form)**

Videos 12 and 13:

https://www.youtube.com/watch?v=FvS7v6KM1ig **(Factoring Polynomials Using GCF)**

https://www.youtube.com/watch?v=Y1Vd5wb0rnM (**More Examples of Factoring Polynomials Using GCF)**

Video 14 and 15:

https://www.youtube.com/watch?v=-4jANGlJRSY **(Factoring $x^2 + bx + c$)**

https://www.youtube.com/watch?v=Nj9GgAM-dTQ **(More Examples of Factoring $x^2 + bx + c$)**

Video 16 and 17:

https://www.youtube.com/watch?v=sjW1nGGXQ9I **(Factoring $ax^2 + bx + c$)**

https://www.youtube.com/watch?v=V3uzxe2JNW4 **(More Examples of Factoring $ax^2 + bx + c$)**

Video 18 and 19:

https://www.youtube.com/watch?v=wmnd4fwxSus **(Factoring Special Products)**

https://www.youtube.com/watch?v=6QQJoDshUt8 **(More Examples of factoring Special Products)**

## Part B : Vocabulary, Hints and Explanations

## Important Vocabulary That Students Need to Understand!

| | |
|---|---|
| monomial | a prefix that means one, therefore monomial is an expression in Algebra that contains one term. Monomials include numbers, whole numbers, and variables.<br>• Monomials must have whole number exponents<br>• Monomials cannot have variables in the denominator.<br>• Monomials cannot have variables as an exponent. |
| Degree of monomial | the sum of the exponents of the variables<br>Ex: $5x^2$ The sum of the exponent of the variables is 2, the degree of the monomial is 2.<br>$5xy^3$ The sum of the exponents of the variables is 3+1 = 4, the degree of the monomial is 4.<br>(remind students that there is a little invisible exponent of 1 when no exponent is present for a variable)<br>5 The sum of the exponents of the variable (what variable? There is none) is zero. The degree of the monomial is 0. |
| polynomial | a monomial (see definition of a monomial) or a sum of monomials. Each monomial (or chunk) is a term in the polynomial. |
| binomial | a prefix that means two, binomial is a polynomial with 2 terms<br>Ex: $8x + x^2$ or $5x + 3$ |
| trinomial | a prefix that means three, trinomial id a polynomial with 3 terms |

| | |
|---|---|
| | Ex: $x^2 + 5x + 2$ |
| standard form | exponents of the variables are written left to right from largest to smallest. Students need to write polynomials in standard form to determine the degree of the polynomial. This also represents "working order" or "working format" when solving polynomials. |

## Adding and subtracting polynomials:

Students should be familiar with combining like terms, which is basically what adding polynomials is!

Students may confuse terms such as $x$ , $x^2$ , and $x^3$ as like terms. When a student has trouble at this level, check the student's understanding of "like terms".

For subtraction, some students can change subtraction to addition and distribute the subtraction sign while others find it easier to use a calculator and subtract.

## Multiplying polynomials:

There is more than one method to use when multiplying polynomials. Begin by multiplying binomials. The strategies apply to trinomials as well.

**Method #1:**

**Multiplying binomials using the distributive property:**

Ex: $(x + 2)(x + 5)$

Distribute the first term to the second by: $x(x + 5) + 2(x + 5)$

Distribute each of these: $x(x) + x(5) + 2(x) + 2(5)$

Multiply: $x^2 + 5x + 2x + 10$

Combine like terms $x^2 + 7x + 10$

**Multiplying binomial times a trinomial using the distributive property:**

Ex: $(x + 2)(x^2 + 5x - 3)$

Distribute first term to the second term: $x(x^2 + 5x - 3) + 2(x^2 + 5x - 3)$

Distribute each of these(some students can skip this step)

$x(x^2) + x(5x) + x(-3) + 2(x^2) + 2(5x) + 2(-3)$

Multiply: $x^3 + 5x^2 + -3x + 2x^2 + 10x + -6$

Combine like terms: $x^3 + 7x^2 + 7x + -6$

**Method #2**

**Multiplying binomials using a table (or box method)**

Ex: $(x + 2)(x + 5)$

Create a box and multiply just as you would on a multiplication table (or a Punnett Square)

| ~~~~~~~~ | x | 2 |
|---|---|---|
| x | $x^2$ | 2x |
| 5 | 5x | 10 |

Combine the terms in the box (red): $x^2 + 7x + 10$

**Multiplying binomials by a trinomial using a table (or box method)**

Create a box and multiply as above:

Ex: $(x + 2)(x^2 + 5x - 3)$

| ~~~~~~~ | $x^2$ | 5x | -3 |
|---|---|---|---|
| x | $x^3$ | $5x^2$ | -3x |
| 2 | $2x^2$ | 10x | -6 |

Combine terms in the box (red): $x^3 + 7x^2 + 7x - 6$

**Method #3**

**Multiplying binomials using FOIL method**

Foil is a mnemonic that stands for:

**F**irst, **O**uter, **I**nner, **L**ast

Ex: (x + 2) (x +5)

Multiply the **F**irst term: (x)(x) = $x^2$

Multiply the **O**uter terms: (x)(5) = 5x

Multiply the **I**nner terms: (2)(x) = 2x

Multiply the **L**ast terms: (2)(5) = 10

Add the products: $x^2 + 5x + 2x + 10$

Combine like terms: $x^2 + 7x + 10$

## Factoring Polynomials:

**Factoring a binomial:** A student finds a common factor

Ex: $x^2 + 2x$ factors as $x(x + 2)$

**Factoring a trinomial:** A student must find the factors of the third term that when added equal the sum of the second term. Some students use a large X with the third term in the top of the X and the sum in the bottom of the X. They work with factors of the third term and place the appropriate set in the sides of the X.

Ex: $x^2 + 5x - 24$ Factors of 24 are 1, 24

2, 12

3, 8

4, 6

The factors 3, 8 have a difference of 5.

One of the factors must by negative for the product to be –24.

The sum of 8 + -3 = 5

$x^2 + 5x - 24$ factors as (x+8) (x-3)

Using any of the methods taught to multiply binomials, the student can check the factors to determine if he has found the correct factors.

As a student works factoring trinomials, he may note that if the third term is positive the factors must have the same sign. Both can be positive or both can be negative for the product to be positive.

If the second term is negative (and the third term is positive), then both factors are negative.

If the third term is negative one factor must be positive and one factor must be negative. If the second term is positive the larger factor is positive. If the second term is negative the larger term is negative.

## Solving Polynomials

To solve the **ROOTS** of the polynomial the student sets the factored form of the polynomial to zero.

**Solving a binomial:**

Ex: $x^2 + 2x$ factored as $x(x + 2)$

Set to zero $x = 0$ $x + 2 = 0$

The roots are 0 and -2

**Solving a trinomial:**

Ex: $x^2 + 5x - 24$ factored as $(x + 8)(x - 3)$

Set to zero $x + 8 = 0$ and $x - 3 = 0$

The roots are - 8 and 3

## Factoring Out a GCF (Greatest Common Factor)

A student may need to factor out a GCF before factoring a polynomial (trinomial).

Ex: $5x^2 + 15x + 10$

Factor out the GCF of 5: $5(x^2 + 3x + 2)$

Then factor the trinomial as explained above: $5(x + 2)(x + 1)$

## Factoring the Difference of Perfect Squares

The difference of perfect square will factor as a "plus/minus" of the second term

Ex: $x^2 - 25$ (both $x^2$ and 25 are perfect squares)

The factors are $(x - 5)(x + 5)$

If the student uses FOIL to check, he will see that the Outers and Inners cancel. $(-5x) + (5x) = 0$

Remind students that 1 is a perfect square in factoring polynomials!

Ex: $x^2 - 1$ factors as $(x - 1)(x + 1)$

## Factoring by Grouping

To factor polynomials with four terms, group the terms into pairs. Factor out the GCF from each pair as needed. Look for a common binomial factor and factor that out of the polynomial.

This is easier to understand with examples!

Ex #1: $x^3 + 3x^2 + 2x + 6$

Group in pairs with common factors $(x^3 + 3x^2) + (2x + 6)$

Factor out the GCF from each pair $x^2(x + 3) + 2(x + x + 3)$

Factor out $(x + 3)$ $(x + 3)(x^2 + 2)$

Ex #2: $x^3 + 2x^2 + 5x + 10$

Group in pairs with common factors: $(x^3 + 2x^2) + (5x + 10)$

Factor out the GCF in each pair $x^2(x + 2) + 5(x + 2)$

Factor out $(x + 2)$ $(x + 2)(x^2 + 5)$

NOTE: Terms can be rearranged to group terms with common factors!!

Ex: $x^3 + 7 + x^2 + 7x$ can be rearranged to group common factors.

Rearrange: $x^3 + x^2 + 7x + 7$

Group: $(x^3 + x^2) + (7x + 7)$

Factor: $x^2(x + 1) + 7(x + 1)$

Factor out: $(x + 1)(x^2 + 7)$